Handout 7 (Writing): Mathematical Writing Exercises

Most of the exercises below are taken from the following manuscripts:

- Vivaldi, Franco. Mathematical Writing. Springer, 2014.
- Gillman, Leonard. Writing mathematics well: a manual for authors. Mathematical Assn of Amer, 1987.
- Halmos, Paul R. How to write mathematics. Enseign. Math 16, no. 2 (1970): 123-152.
- Knuth, Donald Ervin, Tracy Larrabee, Paul M. Roberts, and Paul M. Roberts. Mathematical writing. Vol. 14. Washington, DC: Mathematical Association of America, 1989.
- Lee, Kevin P. A guide to writing mathematics. Retrieved September 12 (2010): 2010.

Exercises

Identify problems with the use of mathematics in the following excepts of text, and rewrite them to fix the problems.

1. Let $f(t)$ be Gryffindor's score t minutes into a game against Slytherin. f is globberfluxible at $t = 3$.

2. The function $z^2 + 1$ is even.

3. On a compact space every real-valued continuous function f is bounded.

4. If $0 \leq \lim_{n\to\infty} \alpha_n^{1/n} = \rho \leq 1$, then $\lim_{n\to\infty} \alpha_n = 0$.

5. The union of a sequence of measurable sets is measurable.

6. For invertible X, X^* is also invertible.

7. Let $\delta = \frac{3}{4}\epsilon > 0$. Then ....

8. If $x > 0$, then Euler proved in 1756 that ....

9. Assume $x = 3$. Therefore $2x = 6$.

10. And when $x = -1$ instead, we can see that...

11. Let the angles of the triangle be δ, a_1, and t.

12. ...Consider the quantity $a_1x + a_2y$.

13. Let the number of elephants in the zoo in year n with $n \in [0, \infty)$ be $e(n)$. Suppose the growth rate is $\frac{de}{dn} = 2.5$.

14. Looking at the graph, we can see that the result is true.

15. (Hypotheses: f is continuous, f is differentiable, G is abelian.) ...
By hypothesis, we have . . .

16. Compute $argmax(sin(x))$.

17. Isaac Newton once said "If I have seen further than others, it is by standing upon the shoulders of giants".

18. Let's derive the formula

$$1 + r + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r}, \qquad r \neq 1. \tag{1}$$

Define the LHS of the above equation to be $S_{r,n} = 1 + r + \cdots + r^n$. We compute

$$rS_{r,n} - S_{r,n} = r^{n+1} - 1.$$

If $r \neq 1$, divide both sides by $r - 1$ to obtain

$$S_{r,n} = \frac{r^{n+1} - 1}{r - 1}$$

which is equivalent to (1).

19. Let P be the escaped wombat population (in thousands) t years after 1990 and suppose that

$$P = 0.5(1.12)^t.$$

The wombat population in 1992 is approximately 672. We can see this by setting $t = 2$ and observing that

$$P = 0.5(1.12)^2 = 0.6272 \text{ thousand wombats.}$$

If we want to predict when the wombat population will reach 2000, we set $P = 2$ and solve for t using logarithms.

$$\begin{aligned} 2 &= 0.5(1.12)^t \\ \log 2 &= \log 0.5 + t \log 1.12 \\ t &= \frac{\log 2 - \log 0.5}{\log 1.12} \approx 12.23 \text{ years.} \end{aligned}$$

The wombat population will reach 2000 in the year 2002.

20. (Proof that the functions e^x and e^{2x} are linearly independent over the reals.)
Assume on the contrary that e^x and e^{2x} are linearly dependent. Then there exist constants c_1 and c_2, not both zero, such that

$$c_1 e^x + c_2 e^{2x} = 0 \qquad \text{for all } x.$$

Then $c_1 + c_2 e^x = 0$. Differentiating, we get $c_2 e^x = 0$, so $c_2 = 0$. But then $c_1 = 0$. Thus, $c_1 = c_2 = 0$.

This contradicts the fact that c_1 and c_2 are not both zero; therefore we must reject the assumption that the functions are linearly dependent. Consequently, they are linearly independent.

21. (Proof of the Mean Value Theorem, from Rolle's Theorem.)
Consider a function f which is continuous on $[a, b]$ and differentiable on its interior. Let

$$F(x) = f(x) - f(a) - \frac{f(b) - f(a)}{b - a}(x - a)\,. \tag{2}$$

Then $F(a) = 0$ and $F(b) = 0$ and F is continuous on $[a, b]$ and differentiable in (a, b). Therefore F satisfies the hypotheses of Rolle's theorem, so there is a point z in (a, b) such that $F'(z) = 0$. Taking the derivative of (2) shows that $f'(z) = (f(b) - f(a))/(b - a)$.

22. Consider a set of n isolated vertices, and let P be the probability measure that is uniform on the set of all undirected graphs. Let $M > 0$ be a positive integer, and consider a sequence of graphs $G_1, G_2, \ldots, G_M$ chosen independently from distribution P. Consider the subsequence $G_{\alpha_1}, \ldots, G_{\alpha_k}$ of graphs with at least $2n$ edges, and suppose they have $n_{\alpha_1}, \ldots, n_{\alpha_k}$ edges respectively. What is the probability that the total number of edges in this subsequence is at least $2Mn$?

23. Simplify this definition:

$$A = \left\{ y \in \mathbb{Q} \;:\; y = \frac{x}{x^2 + 1}, \quad x \in \mathbb{Z}, \quad x < 0 \right\}.$$

24. Simplify / improve this definition:

$$z(y_1, y_2, \ldots) = \sum_{i=1}^{\infty} \sum_{y=0}^{y_i - 1} (y + 1) x^{i-1}.$$

Quotes and Notes

Halmos

- My advice about the use of words can be summed up as follows. (1) Avoid technical terms, and especially the creation of new ones, whenever possible. (2) Think hard about the new ones that you must create; consult Roget; and make them as appropriate as possible. (3) Use the old ones correctly and consistently, but with a minimum of obtrusive pedantry.
- Everything said about words applies, mutatis mutandis, to the even smaller units of mathematical writing, the mathematical symbols. The best notation is no notation; whenever it is possible to avoid the use of a complicated alphabetic apparatus, avoid it. A good attitude to the preparation of written mathematical exposition is to pretend that it is spoken. Pretend that you are explaining the subject to a friend on a long walk in the woods, with no paper available; fall back on symbolism only when it is really necessary.
- One place where cumbersome notation quite often enters is in mathematical induction. Sometimes it is unavoidable. More often, however, I think that indicating the step from 1 to 2 and following it by an airy "and so on" is as rigorously unexceptionable as the detailed computation, and much more understandable and convincing. Similarly, a general statement about n X n matrices is frequently best proved not by the exhibition of many a_{ij}'s, accompanied by triples of dots laid out in rows and columns and diagonals, but by the proof of a typical (say 3x3) special case.

Gillman

- I. Use an uncomplicated symbol in place of an elaborate one. II. Discard any symbol that is just plain unnecessary. III. Simplify the mathematical argument itself.
- That's the interesting thing about this kind of editing: once you have simplified the mathematics and the notation so that the argument is easier to follow, you may see a way of simplifying the reasoning still further.

Other remarks Interesting discussion on pedantic notation

- `https://math.stackexchange.com/questions/2505777/abusing-mathematical-notation-are-these-exampl`