Communication in Mathematics

Teaching, Writing and Oral Presentation

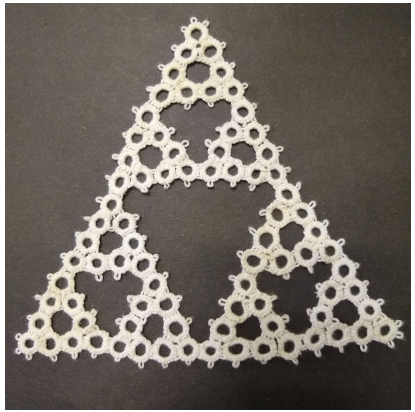
Introductions

Prof. Corrin Clarkson (she / her / her) clarkson@nyu.edu, CIWW 721

Things that bring me joy:

Pearl

Contra Dance



Lace

Introductions

Prof. Corrin Clarkson (she / her / her) clarkson@nyu.edu, CIWW 721

Mathematical Background

- AB University of Chicago
- PhD Columbia University (low dimensional topology)
- Postdoc Indiana University

Teaching Background

- New York City Teaching Seminar
- Inquiry Based Learning Workshop
- MAA Project NExT
- NYU Clinical faculty member

Introductions

Prof. Miranda Holmes-Cerfon (she / her / her) holmes@cims.nyu.edu, CIWW 1107

Things that bring me joy:

Mountains/Hiking/Climbing

Sports / Coaching

Reading

Introductions

Prof. Miranda Holmes-Cerfon (she / her / her) holmes@cims.nyu.edu, CIWW 1107

Mathematical Background

- BSc University of British Columbia
- PhD Courant Institute (geophysical fluid dynamics)
- Postdoc Harvard University

Teaching / Writing Background

- Taught at Harvard, NYU
- Mathcamp Mentor
- cSplash founder
- Interest in style when writing thesis

- Not a professional writer
- But I write professionally
- And I read professionally
- So do you!

Now it’s your turn!

What is this course?

- Half writing, half teaching (oral presentation depending on time)
- Most classes split between the two topics
- Guest lecture from Center for Teaching & Learning on **Feb 12**
- Main references:
  - **Teaching:** MAA Instructional Practices Guide (see website for pdf link)
  - **Writing:** “Style: Lessons in Clarity and Grace”, by Joseph M. Williams & Joseph Bizup
  - Other references & reading to be posted to website
- In-class participation is critical!

Communication

- Website (https://modelingsimulation.github.io/TeachingWriting2020/)
  - Syllabus
  - Schedule
  - Assignments + Due dates
- Google group:
  - communication-in-the-math-SP20-8dbd@nyu.edu
  - You will be automatically signed up for this group
- Resources folder in Google drive
  - You will be automatically signed up for this
- Office Hours

Teaching - Your responsibilities

- Read articles about teaching
- Participate in class discussion
- Observe a class
- Prepare a sample lesson
- Prepare a sample exam

Writing - Your responsibilities

- Introductory paragraph
  - Due before next class
- Research Description
  - ~3 pages
  - Some technical math, some motivation / description
  - Several drafts, that we'll work on throughout the course
- Some reading & exercises outside of class
- Participate participate participate!
  - Share ideas in class
  - Give feedback on other students' writing
  - Many in-class exercises & activities
- Respect for other students; Accept criticism gracefully

Why Writing?

Once upon a time, as a walk through the woods was taking place on the part of Little Red Riding Hood, the Wolf’s jump out from behind a tree occurred, causing her fright.

Once upon a time, Little Red Riding Hood was walking through the woods, when the Wolf jumped out from behind a tree and frightened her.

The adolescents who had effectuated forcible entry into the domicile were apprehended.

We caught the kids who broke into the house.

An understanding of the causal factors involved in excessive drinking by students could lead to their more effective treatment.

We could more effectively treat students who drink excessively if we understood why they do so.

The absence from this dictionary of a handful of old, well-known vulgate terms for sexual and excretory organs and functions is not due to a lack of citations for these words from current literature. On the contrary, the profusion of such citations in recent years would suggest that the terms in question are so well known as to require no explanation. The decision to eliminate them as part of the extensive culling process that is the inevitable task of the lexicographer was made on the practical grounds that there is still objection in many quarters to the appearance of these terms in print and that to risk keeping this dictionary out of the hands of some students by introducing several terms that require little if any elu cidation would be unwise.

-- From the foreword, *Webster's New World Dictionary of the American Language, 2nd Ed, 1972*

This means:

We excluded vulgar words for sex and excretion not because we could not find them. We excluded them because many people object to seeing them. Had we included them, some teachers and schoolboards would have refused to let this dictionary be used by their students, who in any event already know what those words mean.

Introduction

Let $(\xi_k)_{k\in\mathbb{N}}$ be an iid sequence of random variables, on a probability space with a measure P, satisfying $P(\xi_1 = \pm 1) = \frac{1}{2}$. Given $y \in \mathbb{R}$, $h \in (0,\infty)$ and a function $a_h : \mathbb{R} \to \mathbb{R}$, we denote by $(X^h_{kh})_{k\in\mathbb{N}_0}$ the Markov chain defined by

$$X^h_0 = y \quad \text{and} \quad X^h_{(k+1)h} = X^h_{kh} + a_h(X^h_{kh})\xi_{k+1}, \qquad \text{for } k \in \mathbb{N}_0. \tag{1}$$

We choose as the Markov chain's index set the set of non-negative multiples of h because we interpret h as the length of a time step. We extend $(X^h_{kh})_{k\in\mathbb{N}_0}$ to a continuous-time process by linear interpolation, i.e., we set

$$X^h_t = X^h_{\lfloor t/h\rfloor h} + (t/h - \lfloor t/h\rfloor)(X^h_{(\lfloor t/h\rfloor+1)h} - X^h_{\lfloor t/h\rfloor h}), \qquad t \in [0,\infty). \tag{2}$$

Let $\overline{h} \in (0,\infty)$ and let $(a_h)_{h\in(0,\overline{h})}$ be a family of real functions and $(X^h)_{h\in(0,\overline{h})}$ the associated family of extended Markov chains defined as in (2). A fundamental problem of probability theory is to find conditions on $(X^h)_{h\in(0,\overline{h})}$ such that the laws of the processes X^h, $h \in (0,\overline{h})$, converge in some sense as $h \to 0$. In this article we provide an asymptotic condition on the family $(a_h)_{h\in(0,\overline{h})}$ guaranteeing that the laws of the processes X^h, $h \in (0,\overline{h})$, converge as $h \to 0$ to the law of a one-dimensional regular continuous strong Markov process (in the sense of Section VII.3 in [18] or Section V.7 in [19]). In what follows we use the term *general diffusions* for the latter class of processes. Recall that a general diffusion $Y = (Y_t)_{t\in[0,\infty)}$ has a state space that is an open, half-open or closed interval $I \subseteq \mathbb{R}$. We denote by $I^\circ = (l,r)$ the interior of I, where $-\infty \le l < r \le \infty$. Moreover, the law of any general diffusion is uniquely characterized by its speed measure m on I, its scale function and its boundary behavior. Throughout the introduction we

1 Introduction

The purpose of this article is to develop a general theory allowing to formulate, solve and analyse solutions to semilinear stochastic partial differential equations of the type

$$\mathcal{L}u = F(u, \xi)\,, \tag{1.1}$$

where $\mathcal{L}$ is a (typically parabolic but possibly elliptic) differential operator, ξ is a (typically very irregular) random input, and F is some nonlinearity. The nonlinearity F does not necessarily need to be local, and it is also allowed to depend on some partial derivatives of u, as long as these are of strictly lower order than $\mathcal{L}$. One example of random input that is of particular interest in many situations arising from the large-scale behaviour of some physical microscopic model is that of white noise (either space-time or just in space), but let us stress immediately that Gaussianity is *not* essential to the theory, although it simplifies certain arguments. Furthermore, we will assume that F depends on ξ in an affine way, although this could in principle be relaxed to some polynomial dependencies.

Our main assumption will be that the equation described by (1.1) is *locally subcritical* (see Assumption 8.3 below). Roughly speaking, this means that if one rescales (1.1) in a way that keeps both $\mathcal{L}u$ and ξ invariant then, at small scales, all nonlinear terms formally disappear. A "naïve" approach to such a problem is to consider a sequence of regularised problems given by

$$\mathcal{L}u_\varepsilon = F(u_\varepsilon, \xi_\varepsilon)\,, \tag{1.2}$$

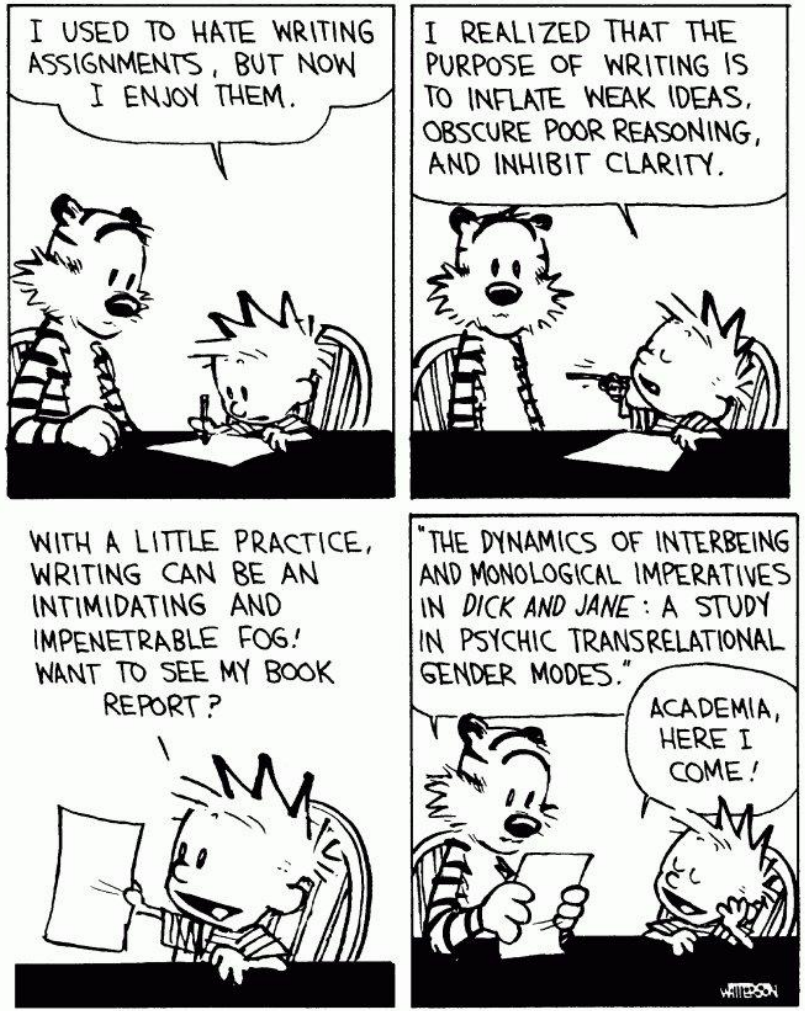
where ξ_ε is some smoothened version of ξ (obtained for example by convolution with a smooth mollifier), and to show that u_ε converges to some limit u which is independent of the choice of mollifier.

This approach does in general fail, even under the assumption of local subcriticality. Indeed, consider the KPZ equation on the line [KPZ86], which is the stochastic PDE formally given by

$$\partial_t h = \partial_x^2 h + (\partial_x h)^2 + \xi\,, \tag{1.3}$$

where ξ denotes space-time white noise. This is indeed of the form (1.1) with $\mathcal{L} = \partial_t -$

Martin Hairer, *A theory of regularity structures,* 2015. (Fields medal work)


I USED TO HATE WRITING ASSIGNMENTS, BUT NOW I ENJOY THEM.
I REALIZED THAT THE PURPOSE OF WRITING IS TO INFLATE WEAK IDEAS, OBSCURE POOR REASONING, AND INHIBIT CLARITY.
WITH A LITTLE PRACTICE, WRITING CAN BE AN INTIMIDATING AND IMPENETRABLE FOG! WANT TO SEE MY BOOK REPORT?
"THE DYNAMICS OF INTERBEING AND MONOLOGICAL IMPERATIVES IN *DICK AND JANE*: A STUDY IN PSYCHIC TRANSRELATIONAL GENDER MODES."
ACADEMIA, HERE I COME!
WATTERSON

Goals

- ~~Inflate weak ideas~~
- ~~Obscure poor reasoning~~
- ~~Inhibit clarity~~

- **Write English clearly**
  - And engagingly?
- Criticize and revise your own work
- Write technical mathematics clearly

You have to have something to say -- this part is up to you!

Why write clearly?

You first!

- Write clearly → Think clearly

You do not really understand something unless you can explain it to your grandmother.
-- (misattributed) Einstein quote

- Extra work for you → Less work for readers
  - → Less time overall
  - → More readers!
  - It's respectful of your readers
- Ideas have a bigger impact
- It's intrinsically satisfying

Feynman was once asked by a Caltech faculty member to explain why spin 1/2 particles obey Fermi-Dirac statistics. He gauged his audience perfectly and said, "I'll prepare a freshman lecture on it." But a few days later he returned and said, *"You know, I couldn't do it. I couldn't reduce it to the freshman level. That means we really don't understand it."*

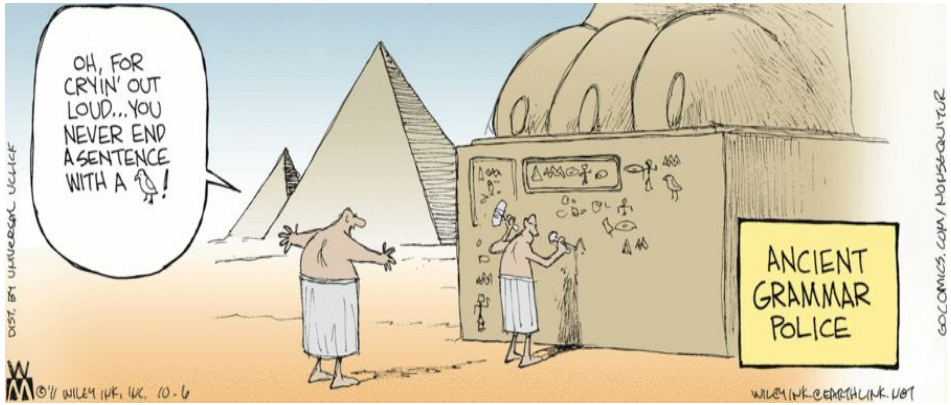
How will we learn to write clearly?

> *Have something to say, and say it as clearly as you can. That is the only secret of style.*
>
> - Matthew Arnold

- There are principles to writing clearly.
  - Not just “Be clear,” “Omit needless words,” “Avoid the passive voice.”
  - Otherwise we could teach soccer just by saying “keep the ball under control” !
- We’ll do exercises to practise each principle (start small, work up)
- We’ll criticise/edit/rewrite each others’ work
- Eventually, principles become ingrained, and you can do this to your own work

Writing - what this is not

- Not a grammar course!



- Not entirely math-dependent
  - Writing is universal, follows the same principles no matter what field
  - We'll look at principles specific to math toward the end of the course
  - We'll use examples from math & science journals
- Won't make writing easy!
  - Writing is hard
  - ∵ Thinking is hard
  - ∵ We can't see our audience

Writing - some vocabulary

- Noun
- Verb
- Adjective
- Nominalization

Noun A person, place, or thing. The kinds of words that refer to things, people, and other nameable or conceivable entities.

Examples: lily, hoax, telephone, bargain, idea, proof, theorem, terror, George Washington, conciousness.

Identify the nouns:

I love watching my cat play with the pink yarn.

It is raining! Everyone, grab your umbrella and rain hat and watch out for the puddles!

Our lack of data prevented evaluation of UN actions in targeting funds to areas most in need of assistance.

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Verb: An action word. The kinds of words which are inflected for tense and which often refer to an action or a state.

Examples: He kicked the football. I thought I saw a pussycat. I am strong.

Identify the verbs:

Once upon a time, as a walk through the woods was taking place on the part of Little Red Riding Hood, the Wolf's jump out from behind a tree occurred, causing her fright.

Once upon a time, Little Red Riding Hood was walking through the woods, when the Wolf jumped out from behind a tree and frightened her.

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Identify the verbs:

The Federalists' argument in regard to the destabilization of government by popular democracy was based on their belief in the tendency of factions to further their self-interest at the expense of the common good.

The Federalists argued that popular democracy destabilized government, because they believed that factions tended to further their self-interest at the expense of the common good.

Identify the verbs:

The Federalists' argument in regard to the destabilization of government by popular democracy was based on their belief in the tendency of factions to further their self-interest at the expense of the common good.

The Federalists argued that popular democracy destabilized government, because they believed that factions tended to further their self-interest at the expense of the common good.

Adjective A descriptive word. Typically refers to a property or state. Often it is a word you can put “very” in front of (but not always; exceptions include “major”, “additional”, etc.)

Examples: big, round, green, afraid, gratuitous, hesitant, occupational.

Identify the adjectives:

The coal mines are dark and dank.

This house is bigger than that one.

Many stores have already begun to play irritating Christmas music.

I love that really big old green antique car.

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Changing verbs & adjectives into nouns, and vice-versa

Nominalization: a noun derived from a verb or an adjective.

Example: nominalize (verb) → nominalization

Verb	→	Nominalization
discover	→	discovery
resist	→	resistance
react	→	reaction

Adjective	→	Nominalization
careless	→	carelessness
different	→	difference
proficient	→	proficiency

Exercise

Turn verbs and adjectives into nominalizations, and nominalizations into adjectives and verbs.

analysis	believe	attempt	conclusion	evaluate
suggest	approach	comparison	define	discuss
expression	failure	intelligent	thorough	appearance
decrease	improve	increase	accuracy	careful
emphasize	explanation	description	clear	examine

Exercise

Now, choose a few verbs & adjectives and create a sentence from them. Then, rewrite this sentence using the corresponding nominalizations. (Do this 3 different times.)

Example: for *suggest, discuss,* and *careful*, you could write:

I *suggest* that we *discuss* the issue *carefully*.

Then rewrite it using nominalizations as

My *suggestion* is that our *discussion* of the issue be done with *care*.

analysis	believe	attempt	conclusion	evaluate
suggest	approach	comparison	define	discuss
expression	failure	intelligent	thorough	appearance
decrease	improve	increase	accuracy	careful
emphasize	explanation	description	clear	examine

Teaching: Big Takeaways

Goal: Build skills and perspective in preparation for teaching a course

- Students are individuals, not younger versions of yourself
- Don't reinvent the wheel: how to find and use resources
- Play with the team: when you teach you are part of a big system
- Learning Goals: you have to know what you are trying to do, in order to know if you have succeeded.
- Evidence based practices: don’t just do what worked for you.

Calculus 1 - Course Goals

Topics

- Limits
- Differentiation techniques
- Applications of derivatives
- Fundamental theorem of calculus

Emphasis

- Conceptual understanding
- ~~Theory~~
- Computational skills
- Applications

Did you take this course in college?

Calculus 1 - Student body

Many majors require college credit for Calculus 1

- Stern undergraduate business school
- Medical schools
- Math, Computer Science, Chemistry and Physics majors

Two main routes for getting credit

- College Board Advanced Placement (AP) exam
- Enrolling in Calculus 1 at NYU

Calculus 1 - The Paradox

AP Exam
- Cost $94
- Once a year.

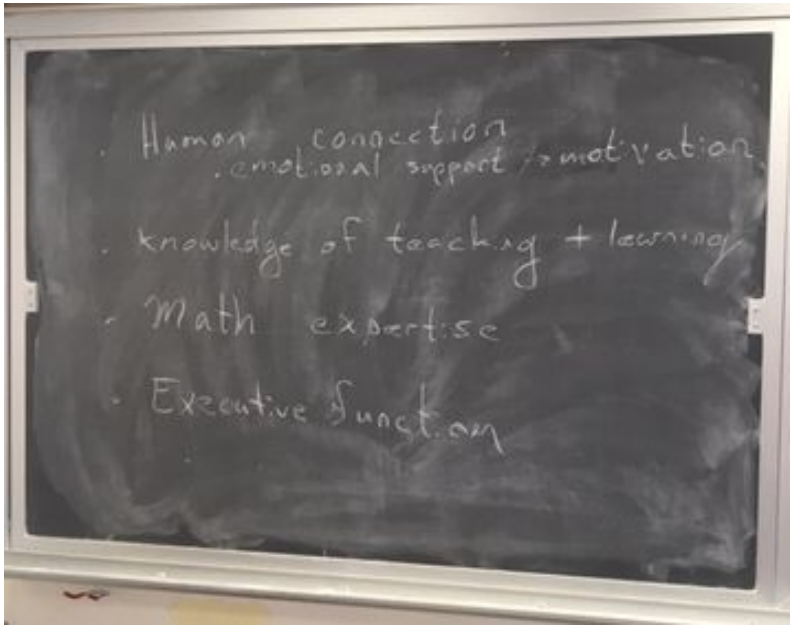
High quality, free resources

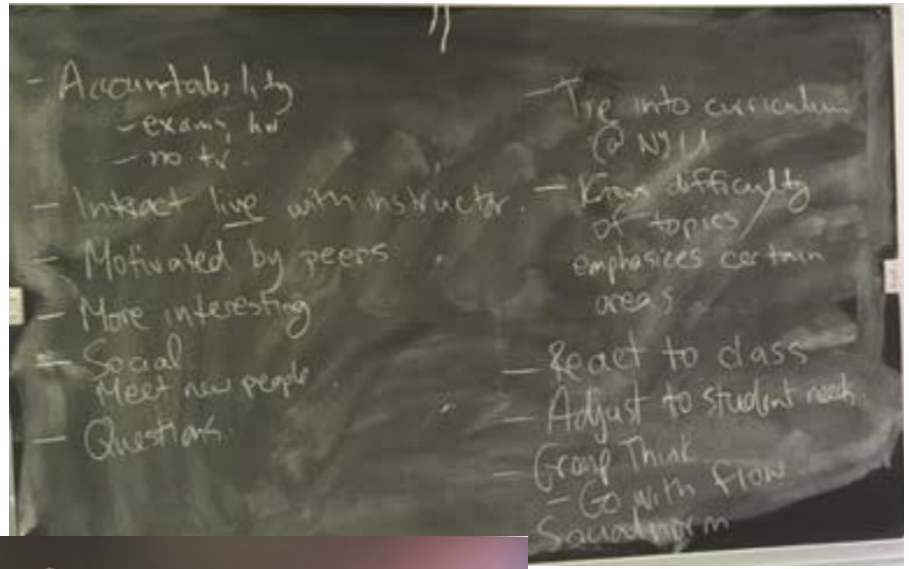
NYU Course
- Cost about $5,700
- Three times a year.

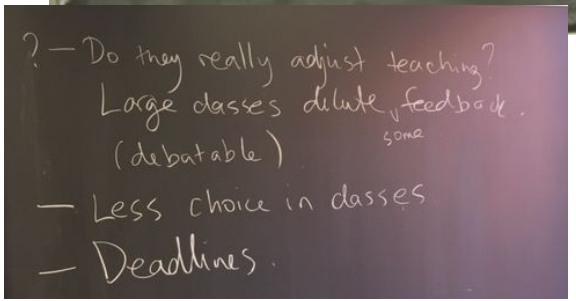
~1,000 NYU students enroll in Calculus 1 each fall

What is the value added by classroom instruction?

Discussion Notes







Teaching Observations

- **Student prior knowledge** - student reflections on pros/cons of writing samples
- **Metacognition** (thinking about thinking) - student reflection on thought process while reading
- **Audience interests** - choosing examples from mathematics papers
- **Wait time** - silence between question prompt and student responses
- **Power dynamics awareness** - students describe importance of writing well before the instructor answers
- **Active Learning** - in class exercises and discussion
- **Teaching Moves**
  - Repeating and rephrasing student contributions
  - Going around the circle
  - Think-pair-share: think quietly, discuss with neighbors, report to big group.
- **Providing emphasis** - de-emphasizing technical grammar issues

Assignments for next week

- Fill out the Welcome Survey (link on Course Schedule) **Due Monday 2/3.**
  - Includes a 1 paragraph (~4-6 sentence) description of your current research problem.
    - Imagine this is content you would post on your website
    - Don’t worry about style! Just write.
- Read MAA Instructional Practices Guide -- Classroom Practices 1 (pg. 1-26) **Due Wednesday 2/5.**
- Send us articles that are particularly well-written or particularly poorly-written (Optional, throughout course)